ECON 289: Problem set 1

Instructor: Ben Brooks

Due date: Tuesday, April 11 at 9:30 AM

1 Programming tasks

1. Install Anaconda Navigator and open Jupyter Notebook.
2. Use Pip to install gurobipy by entering

   ```
   python -m pip install gurobipy
   ```

   in the terminal.
3. Install Gurobi and obtain an academic license.
4. Use Jupyter Notebook to open pset1.ipynb, and follow further instructions.

2 Analytical questions

1. Prove that the dual of the dual is equal to the primal.
2. Consider an LP of the form

   $$\max_{y \in \mathbb{F}^m} cy \text{ s.t. } Ay \leq b, \tilde{A}y = \tilde{b}, \hat{A}y \geq \hat{b}, y_i \geq 0 \ \forall i \in I, y_i \leq 0 \ \forall i \in \tilde{I},$$

   where $I, \tilde{I} \subseteq \{1, \ldots, m\}$. In other words, this LP explicitly includes less than, greater than, and equality constraints. Rewrite this LP in the standard form.

3. Write down the dual of the program in the preceding question. How does the sign restriction on a variable relate to the form of the corresponding dual constraint?

4. Complete the proof of strong duality in the case where $(\hat{x}, \hat{y})$ is a separating hyperplane and $\hat{x}c > 0$.

5. Prove the inequality form of Farkas' lemma via strong duality.

   (Hint: Take the primal objective to be the zero vector.)

6. Suppose that a firm can produce goods $x_1, \ldots, x_n$. There are production technologies $k = 1, \ldots, m$. Technology k produces a bundle $y \cdot b_k \in \mathbb{R}^n$ at a cost of y (meaning that for a cost of y, technology k will produce yb_{ik} units of good i for each i). The firm needs to fill an order $\overline{x}$.

   (a) Write down the LP corresponding to the firm's cost minimization problem.

   (b) Write down the dual. Try to interpret the dual variables and complementary slackness.

7. Consider the optimization problem

$$\min_{x \geq 0} \max\{c_1 x, c_2 x\} \text{ s.t. } Ax \leq b.$$

   Rewrite this optimization as a linear program in standard form.

   (Hint: Introduce an auxiliary variable z.)